Key :

Name: Student number:

Computational Science 260

Second Midterm Exam

Nov. 30, 1995

Marks

12

1. Write the predicate max(L, M) in Prolog. Here, L is a list of numbers, and max(L, M) must succeed if M is the largest element in the list. Otherwise, the predicate should fail. The predicate should also fail for the empty list.

```
max( [X], X).
max( [X | Tail ], X) :- max (Tail, Z), X > Z .
max ( [ X | Tail ], Z) :- max (Tail, Z), X <= Z.
```

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CH5 2. Let $A = \mathbb{P}\{3\}$. Give A is roster notation, and find $\#A$.

$A = \{ \{\}, \{3\} \}$

$\# A = 2$

$\mathbb{P}$ Powerset

$\# A$ Cardinality

$A = \{ \emptyset, \{3\} \}$ $\# A = 2$

$A = \{ \{\}, \{3\} \}$

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CH6 3. Let $f : X \rightarrow Y$ be a partial function from X to Y. Use appropriate phrases to characterize f under the following conditions

Domain is whole
Range is a subset of Range

(a) $\operatorname{dom} f = X$: total function

(b) $\operatorname{dom} f = X, \operatorname{ran} f = Y$: surjective

(c) $\operatorname{dom} f = X, \operatorname{ran} f = Y, \underline{x \neq y \Rightarrow f(x) \neq f(y)}$: bijective

ONTO

injective

1:1

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CH5 4. Let A be a set of people who have attended party 1. and let B be the set of people who have attended party 2. Furthermore. C be the set of people that have attended both parties. and let D be the set that have one. but not both. Use the normal set opertions. such as union. intersection. etc, to express C and D in terms of A and B.

Tuck $C = A \cap B$

9 Tuck $D = (A \cup B) - (A \cap B)$

5. Two relations R and S are given as follows

CH5

$$R = \{(mary, john), (jane, brent), (lia, paul), (anne, ken)\}$$

$$S = \{(lia, carl)\}$$

Find the set $A = \{(x, y) \mid x \notin \text{dom}\, S \wedge xRy\} \cup S$ in roster notation.

$A = \{(mary, john), (jane, brent), (lia, carl), (anne, ken)\}$

10 A updates all information about lia.

CH6 6. Let S be the sibling relation, that is, $(x, y) \in S$ iff x and y have both parents in common. Let H be the halfsibling relation. that is $(x, y) \in H$ iff x and y share the father or the mother, but not both. Furthermore. let I be the identity relation.

(a) Prove that $S \cup I$ is an equivalence relation by verifying that all the properties required for an equivalence relation are met.

(b) Is $H \cup I$ and equivalence relation? Check all the properties required for a relation to be an equivalence relation. and indicate which ones are met.

a) 1. $S \cup I$ is reflexive because I is

2. $S \cup I$ is symmetric: If x is sibling of y, y is sibling of x

3. $S \cup I$ is transitive.

b) 1. $H \cup I$ is reflexive because I is

2. $H \cup I$ is symmetric

3. $H \cup I$ is <u>not</u> transitive: A halfsibling of a halfsibling is normally not a halfsibling.

① Find Relation
② D/T matrix
③ Calcul

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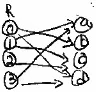
CH 6 7. Let $R : 0..3 \leftrightarrow \{a, b, c, d\}$ be a relation, and let the relation matrix of R be given as follows

TRICKY

$$M_R = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$



$\sim R$
Look backwards
$\{(a,1)\ (a,3)\ (b,0)\ (c,1)\ (c,2)\ (d\ 0)\ (d\ 3)\}$

$R \circ R^\sim \{$ 0-b b-0
0-d d-0
1-a a1, a3
1-c c1 c2
2-c c1 c2
3-a a1 a3
0-d d-3

Give the relation $R \circ R^\sim$ in roster notation.

Do

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array} \odot \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

$m_R \qquad m_{R^\sim}$

in roster notation:

$\{(0,0), (0,3), (1,1), (1,2), (1,3), (2,1), (2,2), (3,0), (3,1), (3,3)\}$

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8. Let *real* be a basic type of Z, and let $x : real$, $y : \mathbb{Z}$. Give the Z declaration of the function f defined by $f(x, y) = x + y$. Here, $x + y$ is evaluated as in Pascal, that is, mixed mode expressions yield a real result.

$f : real \times \mathbb{Z} \rightarrow real$

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9. Consider the following Z fragment which implements a phone directory.

[*name, phone*]

$message ::= ok \mid not\_in\_directory$

$$
\begin{array}{|l}
\hline
\textit{book} \\
directory : name \rightarrow\!\!\!\!\!+\ phone \\
\hline
subscribers = \mathrm{dom}\ directory \\
\hline
\end{array}
\qquad \text{subscriber}
$$

(a) Write two schemas for finding the phone number of a subscriber. The first of these two schemas should apply for the case where the name, call it $x?$, is in the directory, and the second schema should deal with the case where $x?$ cannot be found in the directory.

(b) Suppose the declaration of *directory* is changed from $directory : name \rightarrow\!\!\!\!\!+\ phone$ to $directory : name \leftrightarrow phone$. Write a schema for this case, in which all phone numbers of $x?$ are output.

a)

$$
\begin{array}{|l}
\hline
\textit{find\_number} \\
\Xi\ book \\
number! : phone \\
x? : name \\
confirmation! : message \\
\hline
x? \in subscriber \\
number! = directory\ x \\
confirmation! = ok \\
\hline
\end{array}
\qquad
\begin{array}{|l}
\hline
\textit{not\_listed} \\
\Xi\ book \\
x? : name \\
confirmation! : message \\
\hline
x? \notin subscriber \\
confirmation! = \\
\quad not\_in\_directory \\
\hline
\end{array}
$$

b)

$$
\begin{array}{|l}
\hline
\textit{find\_numbers} \\
\Xi\ book \\
numbers! : \mathbb{P}\ phone \\
x? : name \\
confirmation! : message \\
\hline
numbers! = \{ y : phone \mid x? \mapsto y \in directory \} \\
\hline
\end{array}
$$

Also possible:

$numbers! = directory\ (\!|\ \{x?\}\ |\!)$.